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Polarimetric subtraction
for obtaining the Mueller matrices of components which appear combined in a whole material sample under measurement

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The model

Coherency matrix model for polarimetric phenomena

- Coherency matrix $\mathbf{H}$ (nD)
  - n = 2 *light-2D*
  - n = 3 *light-3D*
  - n = 4 *material*

Generalized Degree of Purity

$$P_n = \left\{ \frac{1}{n-1} \left[\frac{n Tr\left(\mathbf{H}^2\right)}{(Tr\mathbf{H})^2} - 1 \right] \right\}^{\frac{1}{2}}$$

- $\mathbf{H}$ semidefinite positive hermitian matrix

$$\mathbf{H} = \frac{1}{n} \sum_{i=0}^{n-1} m_i \mathbf{E}_i$$

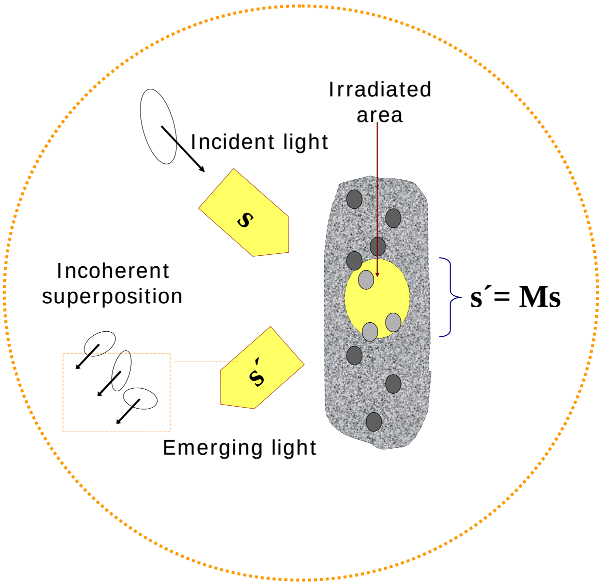
- Trace-orthogonal hermitian matrices basis $\mathbf{E}_i$: SU(n) generators + Identity matrix
- The coefficients of the expansion are the physical quantities:
  - 2D *Stokes parameters*
  - 3D *generalized Stokes parameters*
  - 4D *Mueller parameters*

$$m_i = Tr\left(\mathbf{E}_i \mathbf{H}\right)$$

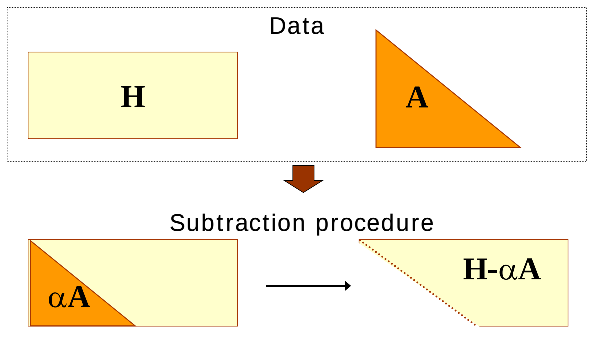
n = 4

Dirac matrices basis	$\mathbf{E}_{kl}$
Expansion of $\mathbf{H}$	$\mathbf{H} = \frac{1}{4} \sum_{k,l=0}^{3} m_{kl} \mathbf{E}_{kl}$
Mueller elements	$m_{kl} = Tr\left(\mathbf{E}_{kl} \mathbf{H}\right)$

The problem



- **H**(**M**) is the coherency matrix of the whole material sample
- **A** is the coherency matrix of the pure element (spatially homogeneous non-depolarizing) suspected to be in the sample
- α is the unknown proportion of **A** in **H** to be calculated by means of the subtraction method



The solution

Case H regular:

- $\mathbf{H}^{-1}\mathbf{A}$ has only one non-null eigenvalue λ
- $\alpha = \frac{1}{\lambda} > 0$
- **H** - α **A** is the coherency matrix of the incognita

Case H singular:

- The subspace generated by the non-null eigenvalues of **H** contains the subspace generated by the non-null eigenvalue of **A**. The subtraction is possible.

  1. Simultaneous zeros-framing of **H** and **A**
  2. Restriction of **H** and **A**

- The subspace generated by the non-null eigenvalues of **H** does not contain the subspace generated by the non-null eigenvalue of **A**. The subtraction is not possible.

Application

- Polarimetry: Mueller matrix measurement, Coherency matrix **H**(**M**) calculation
- Calculation of the Degree of Purity

$$P = \frac{1}{\sqrt{3}}\left(\frac{4Tr\left(\mathbf{H}^2\right)}{(Tr\mathbf{H})^2} - 1\right)^{1/2}$$

➔If $P = 1$: Spatially homogeneous sample ("pure")

➔If $P < 1$: Spatially heterogeneous sample

- $P < 1$; and a component **A** of the sample is known

  1. Calculation of α
  2. Coherency matrix of the rest $\mathbf{R} = \mathbf{H} - \alpha\mathbf{A}$
  3. Mueller matrix of the rest $\mathbf{M}(\mathbf{R}) = \mathbf{M}(\mathbf{H}) - \alpha\mathbf{M}(\mathbf{A})$