# Optical polarimetry.

## The Group of Optical Polarimetry
## Zaragoza University

J. J. Gil[1], J. M. Correas[2], C. Ferreira[2],
I. San José[3], P. A. Melero[2], J. Delso[4]

[1] ICE, Universidad de Zaragoza
[2] Departamento de Matemática Aplicada, Universidad de Zaragoza
[3] Instituto Aragonés de Estadística
[4] Instituto Tecnológico de Aragón

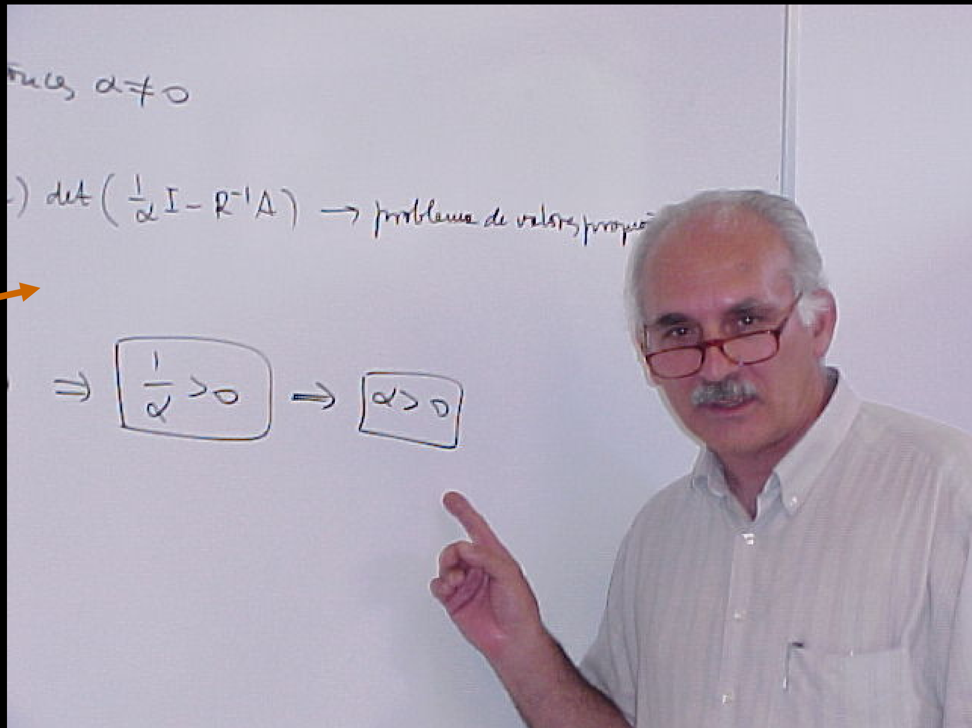
VII Reunión Nacional de Óptica. Santander (Spain) 8-11 Septiembre 2003

# Polarimetry
# Applications and problems to be solved

- *History and interest of this research*
- *Problems to be solved*

# The group

With a piece of the polarimeter




The solution for the polarimetric subtraction was found!

# Goals

- *Obtainment of a proper and complete mathematical model for the adequate representation of all the possible polarimetric behaviors of material media.*
- *Construction of adequate models for the exploitation of the polarimetric measures*

# Results

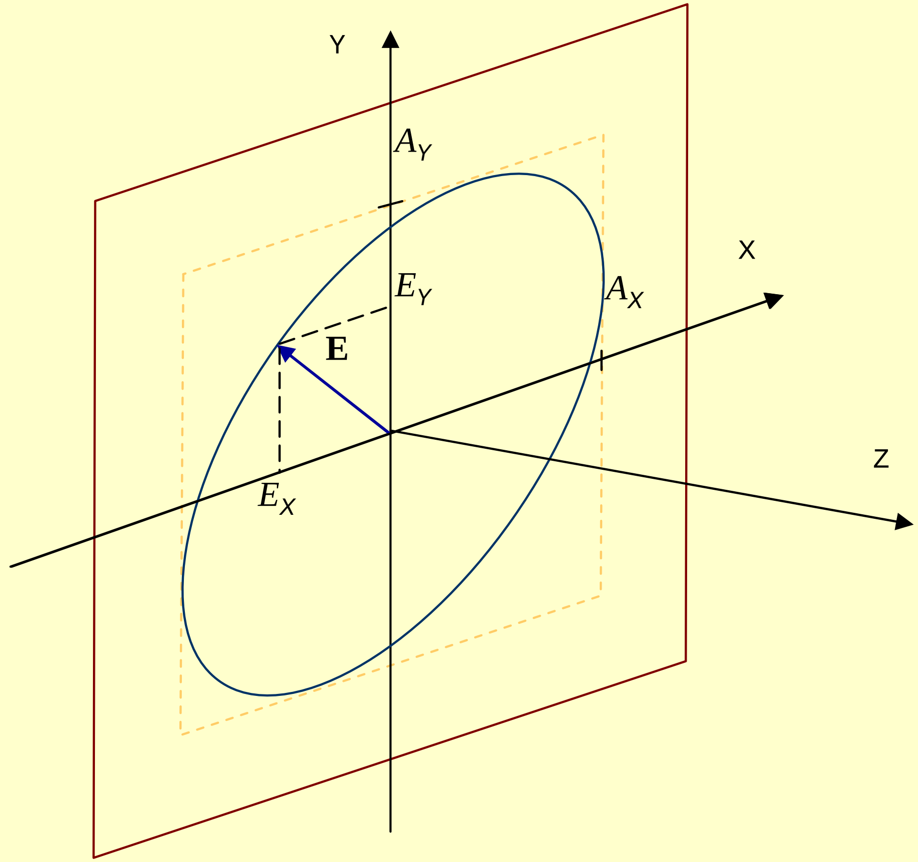
# Polarization algebra:

## Generalized coherency matrix

$$\mathbf{A} = \frac{1}{n}\sum_{i=0}^{n-1} c_i \Theta_i$$

- $n$: order of the matrix
  - ➔ n = *2 2D light. Fixed direction of propagation*
  - ➔ n = *3 3D light. Fluctuating direction of propagation*
  - ➔ n = *4 Polarimetric properties of the medium*
- $c_i$ measurable physical quantities
- $\mathrm{T}_i$ basis of trace-orthogonal Hermitian matrices: generators of SU($n$) group + identity matrix

# Polarized light: 2D case




Elipse de polarización

# Partial polarization:
## Polarization matrix

$$\mathbf{P} = \langle \boldsymbol{\varepsilon}(t) \otimes \boldsymbol{\varepsilon}^{+}(t) \rangle$$

$$\begin{pmatrix} \langle A_x^2(t) \rangle & \langle A_x(t) A_y(t) e^{-i\delta_y(t)} \rangle \\ \langle A_x(t) A_y(t) e^{i\delta_y(t)} \rangle & \langle A_y^2(t) \rangle \end{pmatrix}$$

**positive semidefinite Hermitian matrix**

# Coherency matrix and Stokes parameters

$$\mathbf{P} = \frac{1}{2}\sum_{i=0}^{3} s_i \boldsymbol{\sigma}_i$$

$$s_i = \mathrm{Tr}\left(\mathbf{P}\boldsymbol{\sigma}_i\right),\ i = 0,1,2,3$$

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\quad \boldsymbol{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\quad \boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

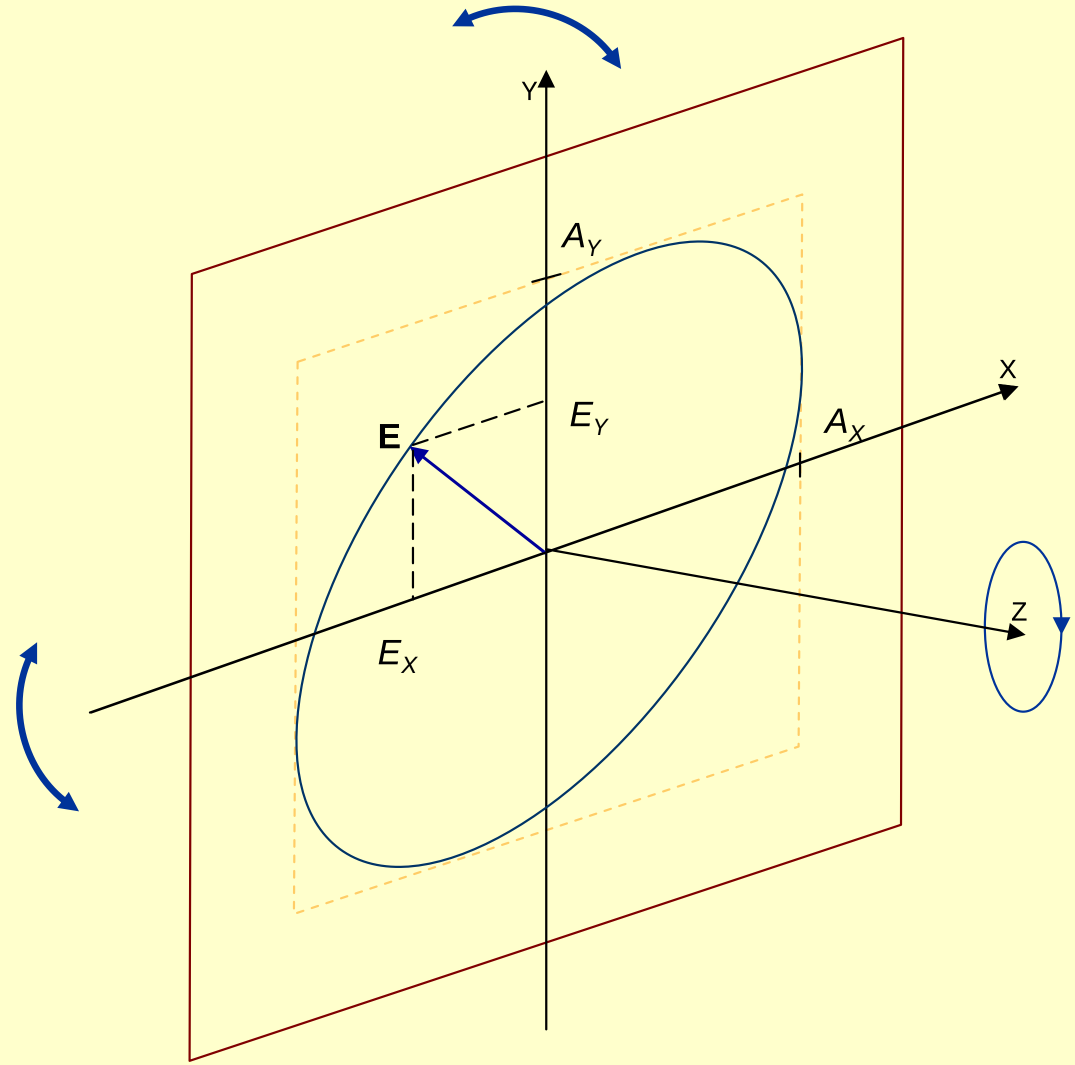
# Degree of polarization

$$G = \left( 1 - \frac{4 Det \mathbf{P}}{\left( Tr \mathbf{P} \right)^2} \right)^{\frac{1}{2}}$$

$$G = \left( \frac{2 Tr \left( \mathbf{P} \right)^2}{\left( Tr \mathbf{P} \right)^2} - 1 \right)^{\frac{1}{2}}$$

$$0 \leq G \leq 1$$

# 3D case: Fluctuating polarization ellipse




Polarization ellipse is inside a changing plane

## Generalized polarization matrix

$$\mathbf{R} = \langle \boldsymbol{\varepsilon}(t) \otimes \boldsymbol{\varepsilon}^{+}(t) \rangle$$

## Generalized Stokes parameters

$$\mathbf{R} = \frac{1}{3} \sum_{i,j=0}^{8} q_j \boldsymbol{\Omega}_i$$

$$q_i = \mathrm{Tr}\left( \mathbf{R}\boldsymbol{\Omega}_i \right), \; i = 0, 1, \ldots, 9$$

# Basis composed of the normalized Gell-Mann matrices plus the identity matrix

$$\Omega_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Omega_1 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Omega_2 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\Omega_3 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Omega_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\Omega_5 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\Omega_6 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\Omega_7 = \sqrt{\frac{3}{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\Omega_8 = \sqrt{\frac{3}{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

# Degree of polarization purity

$$G_{(3)} = \left\{ \frac{1}{2} \left[ \frac{3Tr\left(\mathbf{R}^2\right)}{\left(Tr\mathbf{R}\right)^2} - 1 \right] \right\}^{\frac{1}{2}}$$

# Indices of polarimetric purity

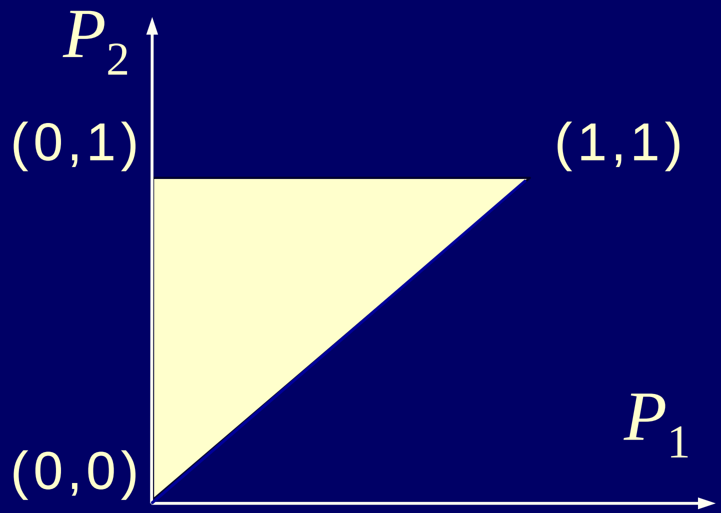
$$P_1 = \frac{\lambda_1 - \lambda_2}{Tr\mathbf{R}}$$

$$P_2 = \frac{\lambda_1 + \lambda_2 - 2\lambda_3}{Tr\mathbf{R}}$$

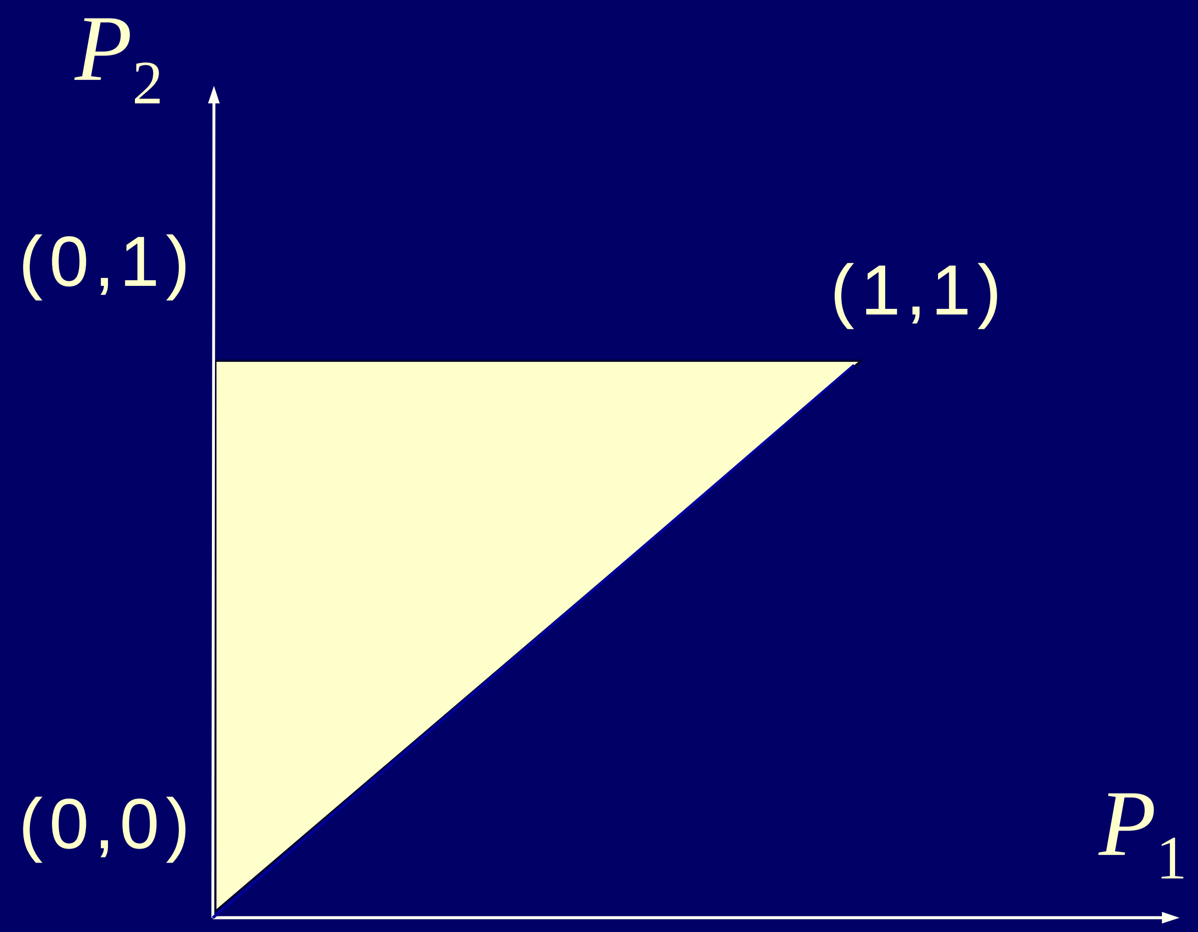
$$0 \le P_1 \le P_2 \le 1$$

$$G_{(3)} = \frac{1}{2}\left[3P_1^2 + P_2^2\right]^{\frac{1}{2}}$$

Región factible de los índices de pureza

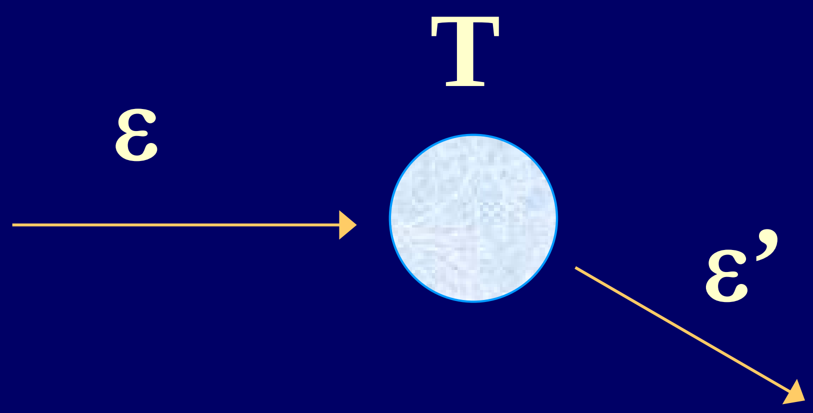


# Purity space



# Polarimetric properties of material media

# Basic interaction




$$\varepsilon' = T\varepsilon$$

$$P' = TPT^{+}$$

# Mueller-Jones matrix

$$\mathbf{N} = \mathbf{L}\left(\mathbf{T} \otimes \mathbf{T}^{*}\right)\mathbf{L}^{-1}$$

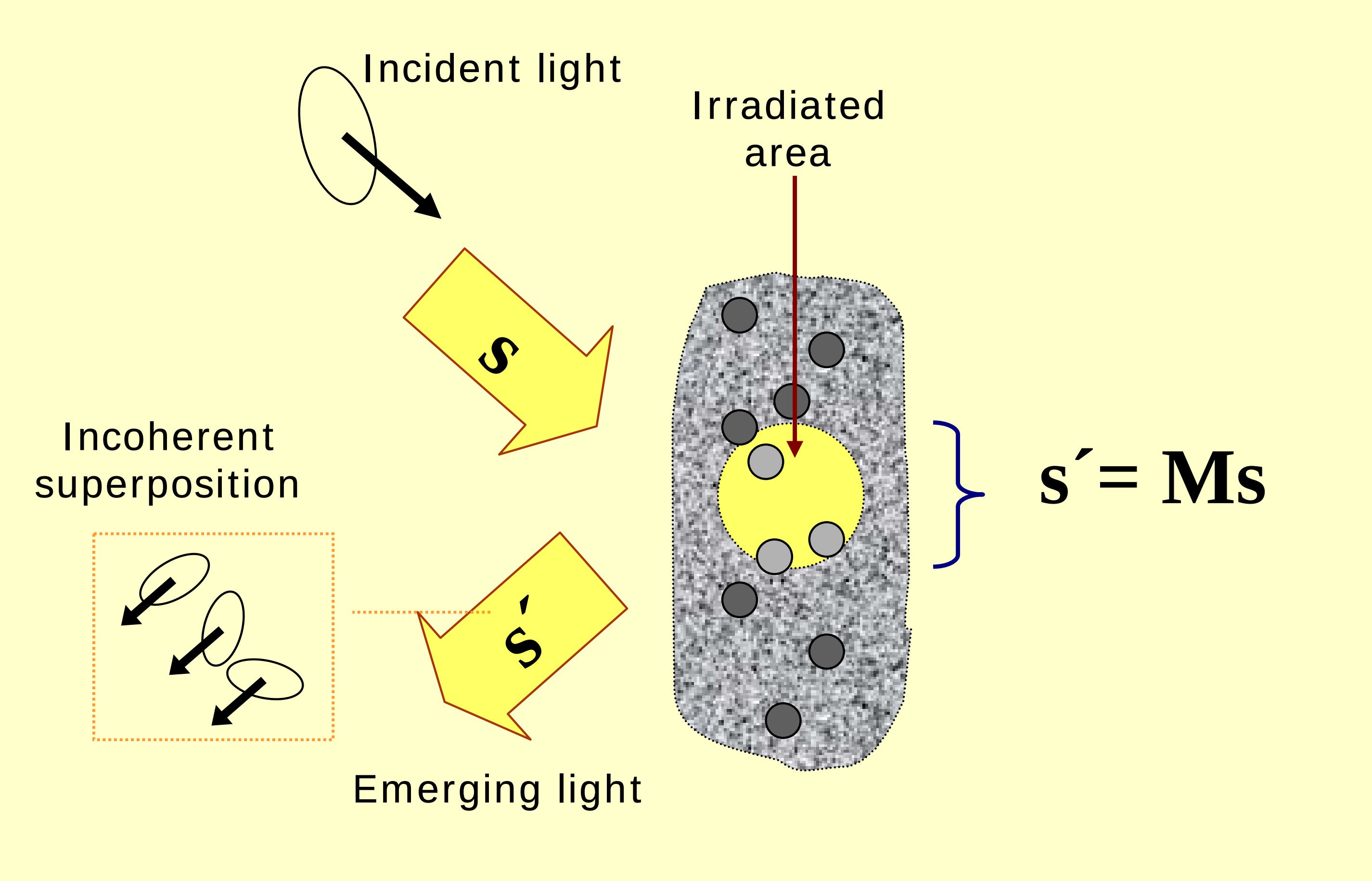
$$n_{kl} = \frac{1}{2} Tr\left(\boldsymbol{\sigma}_{k} \mathbf{T} \boldsymbol{\sigma}_{l} \mathbf{T}^{+}\right)$$

$$k, l = 0, 1, 2, 3$$

$$\mathbf{s}' = \mathbf{N}\mathbf{s}$$

# Mueller matrix

$$\mathbf{M} = \sum_{i}^{n} p_i \left\{ \mathbf{L} \left( \mathbf{T}^{(i)} \otimes \mathbf{T}^{(i)^*} \right) \mathbf{L}^{-1} \right\} = \sum_{i}^{n} p_i \mathbf{N}^{(i)}$$



# Coherency matrix of the medium

$$\mathbf{H} = \frac{1}{4}\begin{pmatrix} \begin{array}{c} m_{00}+m_{01} \\ +m_{10}+m_{11} \end{array} & \begin{array}{c} m_{02}+m_{12} \\ +i\left(m_{03}+m_{13}\right) \end{array} & \begin{array}{c} m_{20}+m_{21} \\ -i\left(m_{30}+m_{31}\right) \end{array} & \begin{array}{c} m_{22}+m_{33} \\ +i\left(m_{23}-m_{32}\right) \end{array} \\ \begin{array}{c} m_{02}+m_{12} \\ -i\left(m_{03}+m_{13}\right) \end{array} & \begin{array}{c} m_{00}-m_{01} \\ +m_{10}-m_{11} \end{array} & \begin{array}{c} m_{22}-m_{33} \\ -i\left(m_{23}+m_{32}\right) \end{array} & \begin{array}{c} m_{20}-m_{21} \\ -i\left(m_{30}-m_{31}\right) \end{array} \\ \begin{array}{c} m_{20}+m_{21} \\ +i\left(m_{30}+m_{31}\right) \end{array} & \begin{array}{c} m_{22}-m_{33} \\ +i\left(m_{23}+m_{32}\right) \end{array} & \begin{array}{c} m_{00}+m_{01} \\ -m_{10}-m_{11} \end{array} & \begin{array}{c} m_{02}-m_{12} \\ +i\left(m_{03}-m_{13}\right) \end{array} \\ \begin{array}{c} m_{22}+m_{33} \\ -i\left(m_{23}-m_{32}\right) \end{array} & \begin{array}{c} m_{20}-m_{21} \\ +i\left(m_{30}-m_{31}\right) \end{array} & \begin{array}{c} m_{02}-m_{12} \\ -i\left(m_{03}-m_{13}\right) \end{array} & \begin{array}{c} m_{00}-m_{01} \\ -m_{10}+m_{11} \end{array} \end{pmatrix}$$

# Coherency matrix and Mueller matrix: General characterization

| | |
|---|---|
| Coherency matrix **H** as an ensemble average | $h_{kj} = \frac{1}{2}\left\langle t_k t_l^* \right\rangle_e$ |
| Basis composed of the modifyed Dirac matrices | $\mathbf{E}_{kl} = \boldsymbol{\sigma}_k \otimes \boldsymbol{\sigma}_l$ |
| Expansion of **H** | $\mathbf{H} = \frac{1}{4}\sum_{k,l=0}^{3} m_{kl}\mathbf{E}_{kl}$ |
| Coefficients of the expansion | $m_{kl} = Tr\left(\mathbf{E}_{kl}\mathbf{H}\right)$ |

# Degree of polarimetric purity of the medium

$$G = \frac{1}{\sqrt{3}}\left(\frac{4Tr\left(\mathbf{H}^2\right)}{(Tr\mathbf{H})^2} - 1\right)^{\frac{1}{2}}$$

$$0 \leq G \leq 1$$

# Generalized degree of purity and purity criteria

$$G_n = \left\{ \frac{1}{n-1} \left[ \frac{n Tr\left(\mathbf{A}^2\right)}{(Tr\mathbf{A})^2} - 1 \right] \right\}^{\frac{1}{2}}$$

$$Tr\left(\mathbf{A}^2\right) \leq (Tr\mathbf{A})^2 \leq n Tr\left(\mathbf{A}^2\right)$$

# Indices of purity

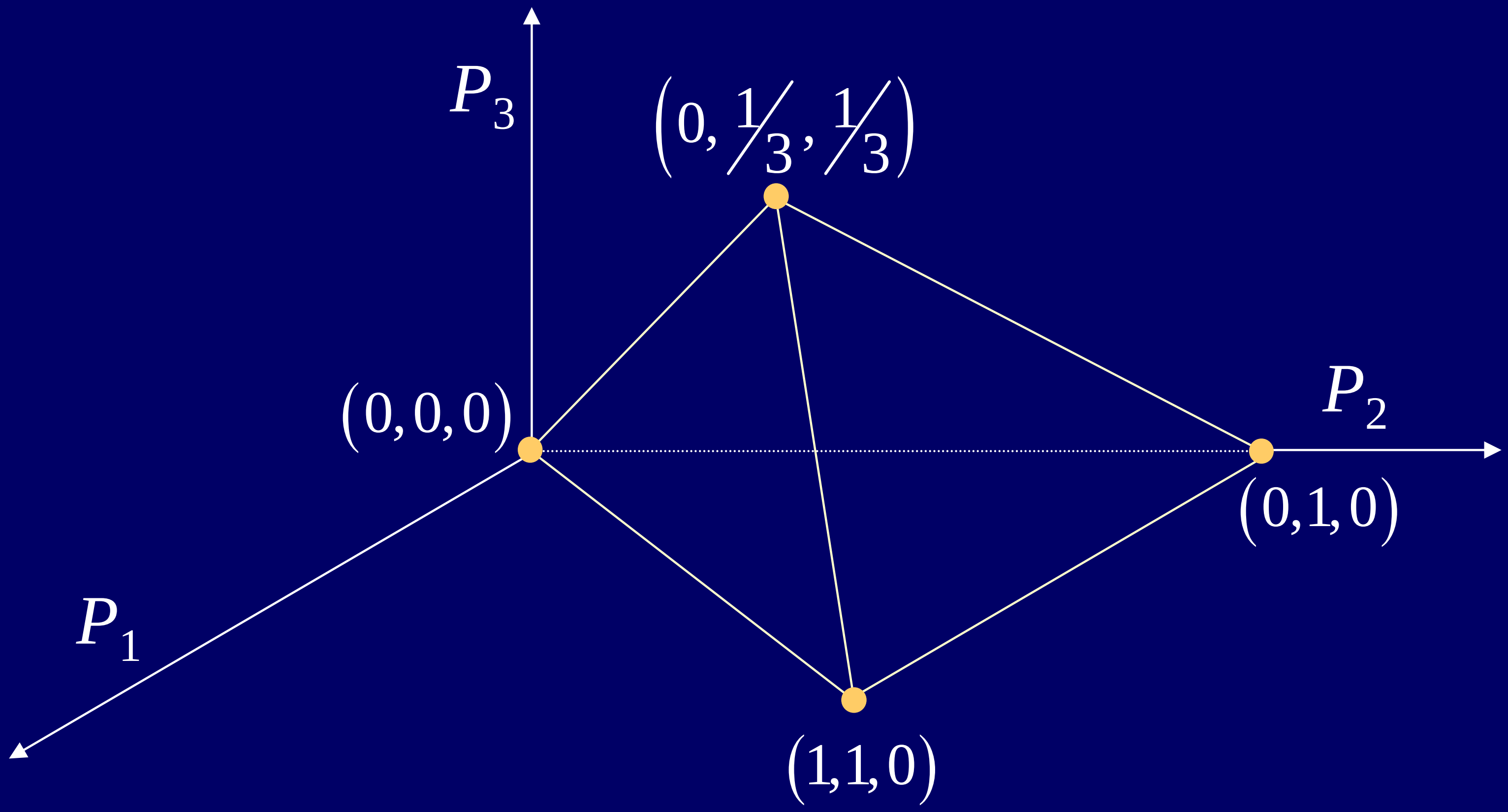
$$P_1 = \frac{\lambda_0 - \lambda_1}{Tr\mathbf{H}}$$

$$P_2 = \frac{(\lambda_0 + \lambda_1) - (\lambda_2 + \lambda_3)}{Tr\mathbf{H}}$$

$$P_3 = \frac{\lambda_2 - \lambda_3}{Tr\mathbf{H}}$$

$$G = \frac{1}{\sqrt{3}}\left(2P_1^2 + P_2^2 + 2P_3^2\right)^{\frac{1}{2}}$$

# Purity space




Región factible de los índices de pureza

# Modeling of the polarimetric properties through coherency matrices

- *Coherency matrix (nD)*
- *Basis of nxn trace-orthogonal Hermitian matrices composed of the generators of the SU(n) group plus the identity matrix*
- *The coefficients of the expansion of the coherency matrix in this basis, are the measurable quantities with physical significance*

# Kernel matrix associated with a Mueller matrix

$\mathbf{M} = \mathbf{RZL}$

- $\mathbf{R}, \mathbf{L}$ *Retarders*
- $\mathbf{Z}$ *Kernel matrix:*
  - **Polarizance-diattenuance**
    - *Forward polarizance*
    - *Reverse polarizance*
  - **Depolarizance**
    - *Indices of purity*

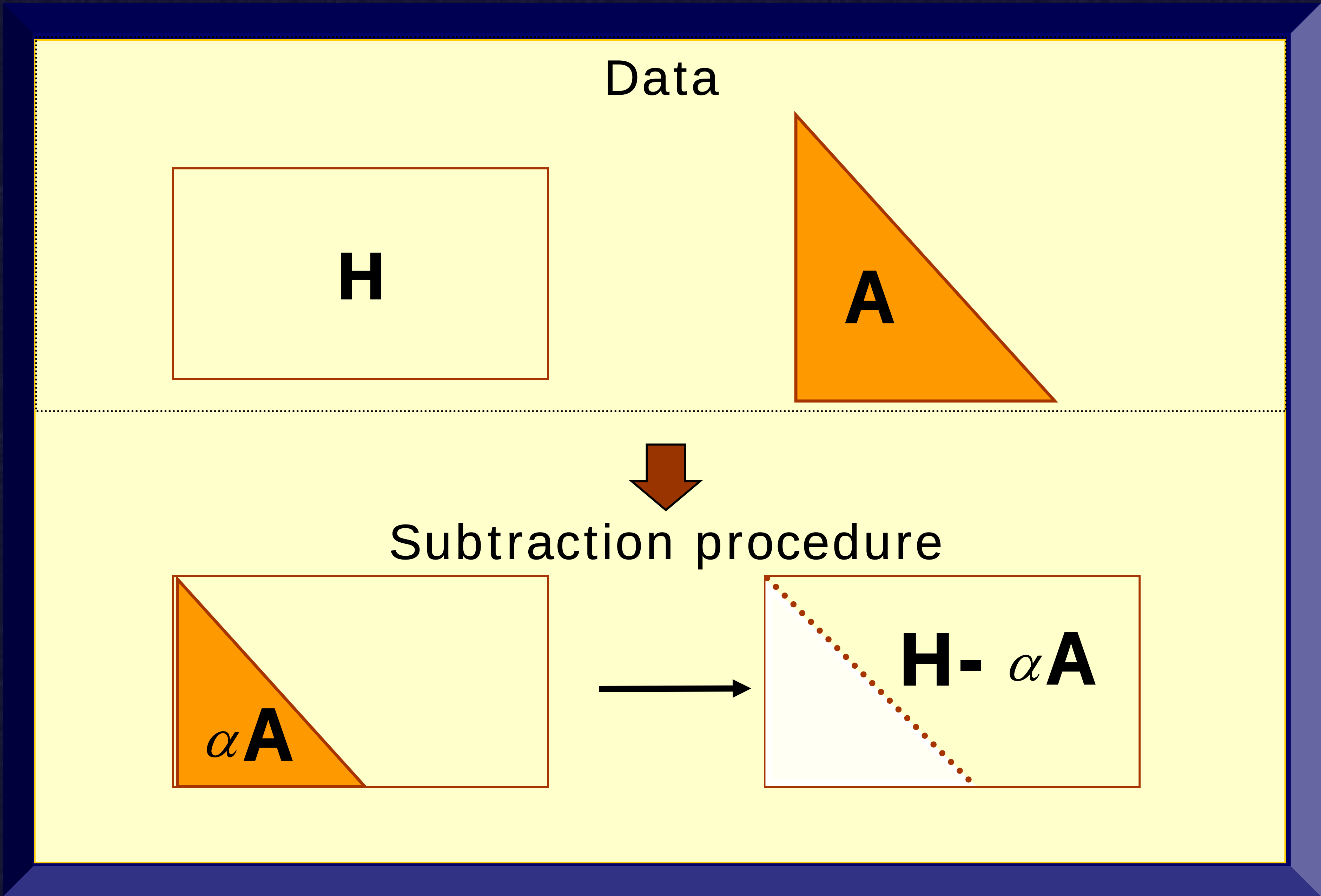
# Decompositions into pure components

$$\mathbf{A} = \sum_{i=0}^{n} p_i \mathbf{X}_i$$

$$\left( \sum_{i=0}^{n} p_i = 1 \right)$$

- $\mathbf{X}_i$ *pure components*
  - **Light:** ***incoherent superposition of totally polarized beams***
  - **Medium:** ***Parallel combination of pure components***

# Polarimetric subtraction



# Overview of the group

- *Collaborators*
- *Citations*
- *Future*