Decomposition of Mueller matrices into pure optical media

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The experimental device

The group

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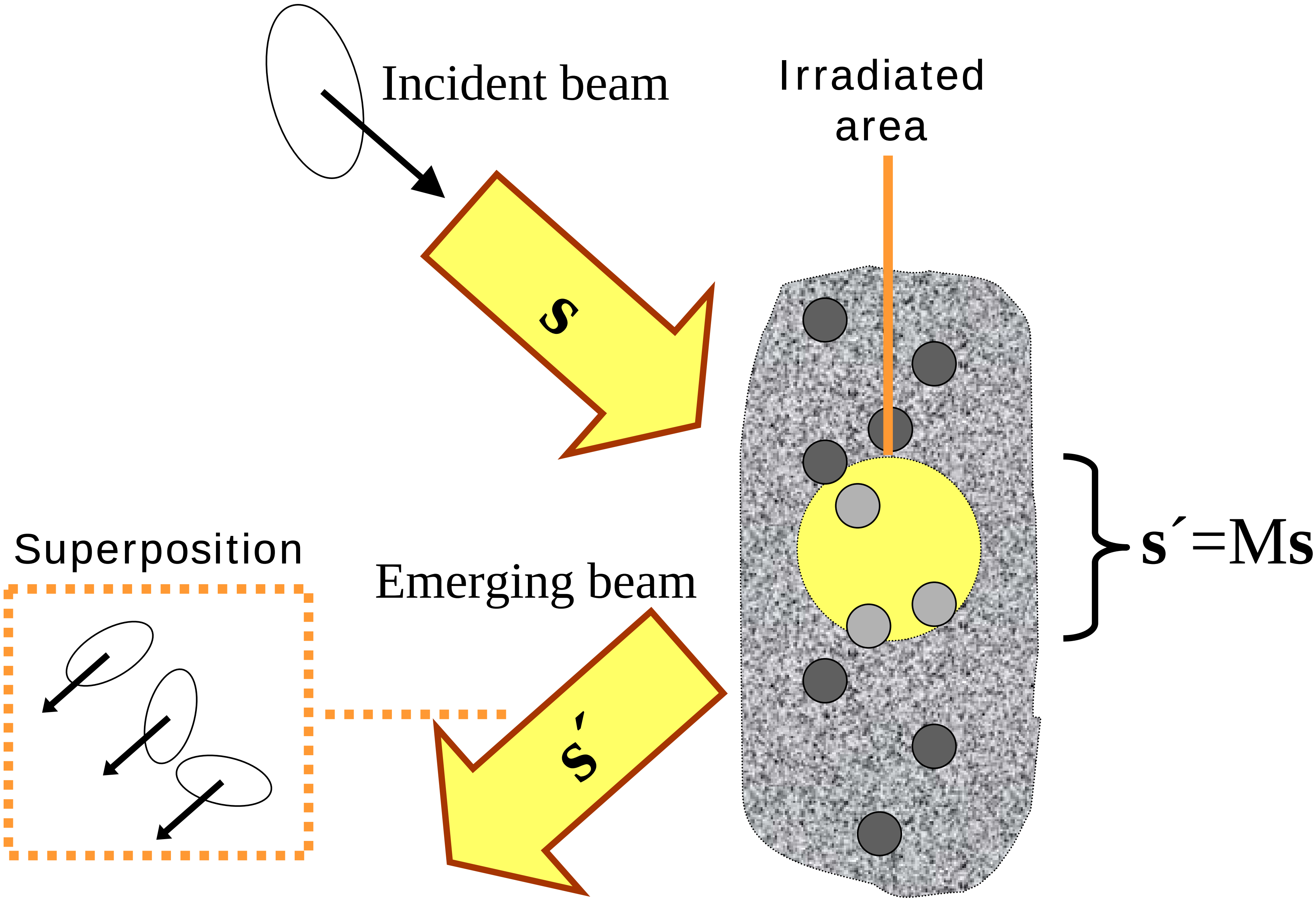
Complex sample



Parallel decomposition of M

$$H = W \Lambda W^{+} =$$

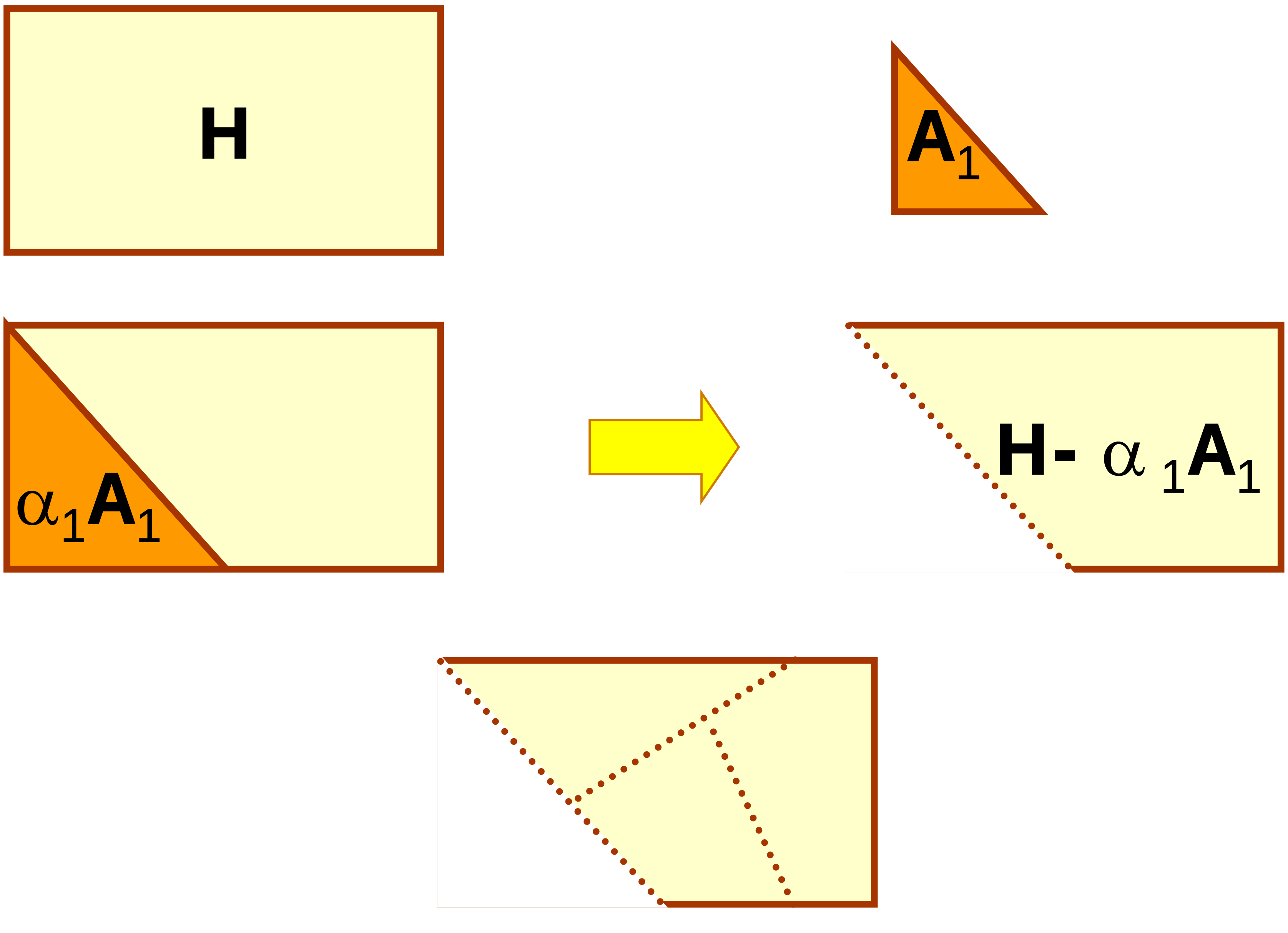
$$= \lambda_0 \, WD(1,0,0,0) W^{+} + \lambda_1 \, WD(0,1,0,0) W^{+}$$

$$+ \lambda_2 \, WD(0,0,1,0) W^{+} + \lambda_3 \, WD(0,0,0,1) W^{+},$$

⇓

$$M = \lambda_0 \, N_0 + \lambda_1 \, N_1 + \lambda_2 \, N_2 + \lambda_3 \, N_3$$

Analysis and decomposition



Main problem

Given n-dimensional hermitic positive semidefined matrices **H, A** *with*

$$rang\mathbf{A} = 1,\ rang\mathbf{H} = r,\ 0 < r \leq n,$$

does exist $\alpha > 0$ *such that*

$$rang(\mathbf{H} - \alpha\mathbf{A}) = r - 1?$$

Algebraic decomposition procedure: regular case

If rank $(\mathbf{H}) = n$, $1/\alpha > 0$ *is the only eigenvalue different from 0 of the matrix* $\mathbf{H}^{-1}\mathbf{A}$

Main problem

Given n-dimensional hermitic positive semidefined matrices **H, A** *with*

$$rang\mathbf{A} = 1,\ rang\mathbf{H} = r,\ 0 < r \leq n,$$

does exist $\alpha > 0$ such that

$$rang(\mathbf{H} - \alpha\mathbf{A}) = r - 1?$$

Algebraic decomposition procedure: general case

If rank (**H**) $= r < n$*, the existence of* $\alpha > 0$ *such that rank*(**H**- α**A**) $= r - 1$ *is not always true. So an efficient test is necessary to be implemented*

Test

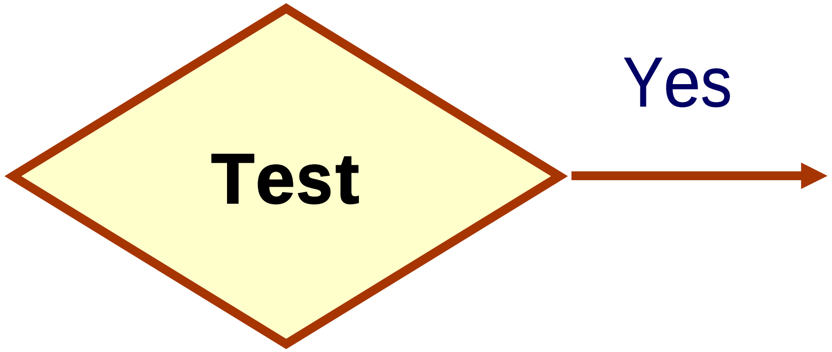
Denoting $\mathbf{A}=(a_1\mathbf{t},\ldots, a_n\mathbf{t})^T$, $\mathbf{t} \neq 0$ *with*
V_1=span{columns of **A**}
V_r=span{rows of **H**}
If $\mathbf{L}= [l_{kj}]$ is a regular matrix such that
$\mathbf{LH}=(\mathbf{v}_1,\ldots, \mathbf{v}_r,\mathbf{0},\ldots,\mathbf{0})^T$
$\mathbf{L}(\mathbf{H}- \alpha\mathbf{A}) = (\mathbf{v}_1-\alpha\lambda_1\mathbf{t},\ldots, \mathbf{v}_r-\alpha\lambda_r\mathbf{t}, -\alpha\lambda_{r+1}\mathbf{t},\ldots,-\alpha\lambda_n\mathbf{t})^T$

where

$$\lambda_k = \sum_{j=1}^{n} l_{kj} a_j \, , \; k = 1(1)n$$

Then $\lambda_i=0$, $i=r+1(1)n$ iff $V_1\subset V_r$ and $\exists\ \alpha>0$ with $\text{rang}(\mathbf{H}- \alpha\mathbf{A})=r-1$

If $\exists i\in[r+1,n]$ with $\lambda_i \neq 0$, then $C^{r+1}=V_1 \oplus V_r$ and $\text{rang}(\mathbf{H}- \alpha\mathbf{A})=r\ \forall\alpha>0$



$$\mathbf{L}\mathbf{H}\overline{\mathbf{L}}^{T} = \begin{pmatrix} \mathbf{H}' & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{L}\mathbf{A}\overline{\mathbf{L}}^{T} = \begin{pmatrix} \mathbf{A}' & 0 \\ 0 & 0 \end{pmatrix}$$

Solve the r-dimensional regular case **H´ , A´**

An application example

Complex | **Pure**

$$\mathbf{H}=\begin{pmatrix} \frac{53}{320} & \frac{3}{64} & \frac{3}{64} & \frac{61}{320} \\ \frac{3}{64} & \frac{53}{320} & \frac{1}{64}-\frac{3i}{20} & \frac{3}{64} \\ \frac{3}{64} & \frac{1}{64}+\frac{3i}{20} & \frac{53}{320} & \frac{3}{64} \\ \frac{61}{320} & \frac{3}{64} & \frac{3}{64} & \frac{77}{320} \end{pmatrix} \qquad \mathbf{P}=\begin{pmatrix} \frac{9}{32} & \frac{3}{32} & \frac{3}{32} & \frac{9}{32} \\ \frac{3}{32} & \frac{1}{32} & \frac{1}{32} & \frac{3}{32} \\ \frac{3}{32} & \frac{1}{32} & \frac{1}{32} & \frac{3}{32} \\ \frac{9}{32} & \frac{3}{32} & \frac{3}{32} & \frac{9}{32} \end{pmatrix} \quad \left(\alpha=\tfrac{1}{2}\right)$$

$$\mathbf{H}-\tfrac{1}{2}\mathbf{P}=\begin{pmatrix} \frac{1}{40} & 0 & 0 & \frac{1}{20} \\ 0 & \frac{3}{20} & \frac{-3i}{20} & 0 \\ 0 & \frac{3i}{20} & \frac{3}{20} & 0 \\ \frac{1}{20} & 0 & 0 & \frac{1}{10} \end{pmatrix} \qquad \mathbf{P}_1=\begin{pmatrix} \frac{1}{8} & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad \left(\alpha=\tfrac{1}{5}\right)$$

$$\mathbf{H}-\tfrac{1}{2}\mathbf{P}-\tfrac{1}{5}\mathbf{P}_1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{3}{20} & \frac{-3i}{20} & 0 \\ 0 & \frac{3i}{20} & \frac{3}{20} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{R}=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{-i}{2} & 0 \\ 0 & \frac{i}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \left(\alpha=\tfrac{3}{10}\right)$$

$$\mathbf{R}_1=\begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{-1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{-1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

The Test answers that the material $\mathbf{R}_1$ is not contained in **H**

Proyecto DGA P064/2000

Diseño de técnicas de análisis de medidas polarimétricas. Su aplicación a la caracterización de materiales

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