# Coherency matrices: a unified model for the mathematical representation of polarimetric phenomena


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## Coherency matrix

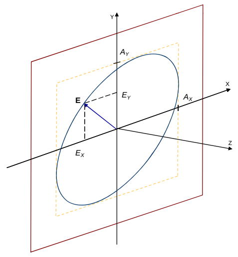
$$\mathbf{H}=\frac{1}{n}\sum_{i=0}^{n-1} m_i \mathbf{E}_i$$

- $n$: order of the matrix
  - $n$ = 2, 2D light. Fixed direction of propagation
  - $n$ = 3, 3D light. Fluctuating direction of propagation
  - $n$ = 4. Polarimetric properties of the material medium
- $m_i$ measurable physical quantities
- $\mathbf{E}_i$ basis of trace-orthogonal Hermitian matrices: generators of the SU($n$) group + identity matrix

## 2D light

$$\mathbf{P}=\frac{1}{2}\sum_{i=0}^{3} s_i \boldsymbol{\sigma}_i$$

- $\mathbf{P}$ Polarization matrix (coherency matrix)
- $s_i$ Stokes parameters
  $s_i = Tr(\boldsymbol{\sigma}_i \mathbf{P})$
- $\boldsymbol{\sigma}_i$ Pauli matrices + identity matrix

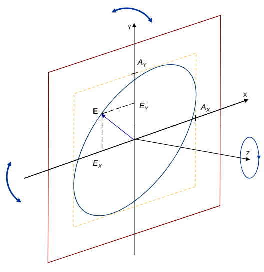


Polarization ellipse placed in a fixed plane

## 3D light

$$\mathbf{R}=\frac{1}{3}\sum_{i=0}^{8} q_i \boldsymbol{\Omega}_i$$

- $\mathbf{R}$ 3D polarization matrix
- $q_i$ 3D Stokes parameters
  $q_i = Tr(\boldsymbol{\Omega}_i \mathbf{R})$
- $\boldsymbol{\Omega}_i$ Gell-Mann matrices + identity matrix

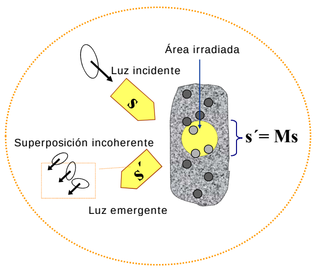


Polarization ellipse placed in a fluctuating plane

## Medium

$$\mathbf{H}=\frac{1}{4}\sum_{i=0}^{8} m_{ij} \mathbf{E}_{ij}$$

- $\mathbf{H}$ 4D Coherency matrix of the medium
- $m_{ij}$ elements of the Mueller matrix
  $m_{ij} = Tr(\mathbf{E}_{ij} \mathbf{H})$
- $\mathbf{E}_{ij} = \boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j$ Modified Dirac matrices




## Polarimetric purity

**Degree of purity**

$$G_n=\left\{\frac{1}{n-1}\left[\frac{nTr\left(\mathbf{H}^2\right)}{(Tr\mathbf{H})^2}-1\right]\right\}^{\frac{1}{2}}$$

Purity interval

$$Tr\left(\mathbf{H}^2\right)\le (Tr\mathbf{H})^2 \le nTr\left(\mathbf{H}^2\right)$$

- $Tr\left(\mathbf{H}^2\right)=(Tr\mathbf{H})^2$ Condition for total purity
- $(Tr\mathbf{H})^2 = nTr\left(\mathbf{H}^2\right)$ Equiprobable mixture of pure states